893. Phytolacca; fructu monopyreno, majore; folio longiore, glabro.
894. Piloſella, major, umbellifera, macrocaulos. *Floribus eſt floſculoſis.* Col. Ecph. 248.
895. Roſa; Pimpinellæ folio, Scotica; flore eleganter variegato.
Roſa Ciphiana. Sibald. Scot. Illuſt.
896. Roſa ſylveſtris, Virginienſis.
897. Senecio Americanus; folio haſtato, nitide ſerrato.
898. Sium umbellatum, repens. Ger. Emac. 256.
899. Solanum, fruticoſum, Africanum; Lauri foliis.
900. Vulneraria erecta annua; folio ſubrotundo leviter crenato.
Loto affinis, Coryli folio. Dod. Mem.

VI. *A Phyſico-mathematical Demonſtration of the Impoſſibility and Inſufficiency of* Vortices: *By M.* de Sigorgne. *Tranſlated from the* French *by* T. S. *M. D.* F. R. S.

THAT natural Philoſophers of an inferior Claſs, who conſider only the Outſide of Things, are obſtinate in the Defence of *Vortices*, is, in my Opinion, not to be wondered at: The Idea of them ſtrikes the Mind very agreeably at firſt, and even ſeems to promiſe the true Mechaniſm. But that Perſons verſed in the moſt profound Geometry, and in the moſt ſublime Calculations, able Academicians, who inceſſantly apply themſelves to the Study of

Nature, ſhould plunge headlong into theſe Notions, and ſuſtain the *Vortices pro aris & focis*, is to me Matter of unaccountable Surprize.

It appears to me, that a *Vortex* is as ſhocking upon mature Conſideration, as its Idea is ſatisfactory upon the firſt mentioning it. And *Vortices*, in my Opinion, are like ſmart Sayings *(bons mots)*, which charm the Mind the firſt, or, perhaps, the ſecond time, but by a Repetition become quite flat and inſipid. What Man, indeed, (if free from Prejudice, and that the Spirit of Party has not depraved his Judgment) would not be aſtoniſhed to ſee brought on the Scene, not only *Vortices*, but ſuch as are compoſed of an infinite Number of ſmaller *Vortices*, each of which ſtill contains an innumerable Number of others ſubordinate to them? For Example: What is this Air, this Water, this Oil, *&c.* which Monſieur *de Moliers* takes pains to introduce? A Sport of the Imagination, or of the Mind, if you pleaſe; but in reality a Paper-building. It has been long ſince ſaid, that according as *Vortices* ſhall be multiplied, they will degenerate into Littleneſs and Puerility: And theſe are the Sentiments even of the good *Carteſians* of our Days. But might it not be ſaid, that the great *Vortices* having the ſame Origin with the little, the latter ſhew the Meanneſs of Extraction of the former? As Matter is diviſible *in infinitum*; as to Vorticity, there is no Difference between the Great and the Small: And conſequently, we have a Right to reject the large *Vortices*, ſince *Carteſians* proſcribe the ſmall.

It is on this Conſideration that I am reſolved to attack the *Vortices*: For I muſt own, to the Shame of our Nation, that the Spirit of Party is ſo predominant therein, that ſeveral Perſons, who by a cloſe Study have found the Inſufficiency of *Vortices* for explaining the *Phænomena* of the Heavens, yet have not dared to publiſh their Notions on that Subject.

But as at preſent the Syſtem of ſmall *Vortices* is freely attacked, I think, as already ſaid, that I have a Right to attack the large; and to this Purpoſe I hope to prove,

I. That the mechanical Formation of a *Vortex* is impoſſible.

II. That the *Vortex*, were it formed, cannot be of long Duration.

III. In fine, that it is not ſufficient for explaining the *Phænomena*.

FIRST PART.

The mechanical Generation of the Vortex *is impoſſible.*

DEMONSTRATION.

In the Hypotheſis of a perfect *Plenum*, GOD at firſt created Matter indefinite, uniform, homogeneous, and at Reſt. This is allowed by all *Carteſians*, and follows in their Principles from this alone, that Matter was created at Reſt. Now, from this perfect Homogeneity of Matter it evidently reſults, in my Opinion, that the *Vortex* cannot be mechanically formed. Suppoſe, ſay the *Carteſians*, that while Matter is as yet at Reſt, GOD imprints a Motion in a ſtrait Line on one of its Particles: This Particle will every Inſtant meet with Obſtacles to the rectilinear Motion in the encompaſſing Matter; this Motion muſt therefore be turned aſide, and will by this means become circular.

But why ſhould the encompaſſing Matter, which is at Reſt, be an Obſtacle to the rectilinear Motion? Becauſe, ſay they, it happens to be in the Line deſcribed by the Particle, on which Motion is ſuppoſed to be imprinted. But this very Reaſon would alſo prove, that the Body ſuppoſed to be in Motion could not circulate round a Centre at a Diſtance from it; becauſe it would conſtantly meet with Matter at Reſt in the Sides of the Polygon which it was to have deſcribed.

In a Word, it is a received Principle, that a Body which moves in a homogeneous Medium, never quits the Line of its firſt Direction: It does not refract, or deviate on one Side or the other of this Direction, except when it paſſes from an eaſier into a more difficult Medium, or from a denſer into a leſs denſe Medium: and even then its Direction muſt be oblique on the Surface of this Medium.

Now, the Body in Queſtion would move in a Medium intirely homogeneous; ſeeing all the created Matter is ſuppoſed to be ſo, and that all but one Particle of this Matter is at Reſt. It is moreover evident, that as all the Matter is uniform, every Direction, of what kind ſoever, of a Body which moves in the midſt of this Matter, will be perpendicular to the Surface which correſponds to it; as is demonſtrated in Mechanics. The ſuppoſed *Mobile* will therefore always move in the Line of its firſt Direction, until it has communicated all its Force; or rather it will remain at Reſt after the leaſt Shock, if Regard be had to nothing more than what I have hitherto ſaid.

But there ſtill remains a very important Remark to be made on this Subject, to wit, that as it is univerſally agreed at this Day, that Reſt is not a Force, all this Matter created at Reſt will be infinitely ſoft: Its Parts will have no Tenacity, no Connexion, no Viſcoſity; they will be but contiguous, and will not have more Adheſion to one another, than Two Globes which would touch out of the Bounds of the World without any reciprocal Attraction; ſince Tenacity, Viſcoſity, *&c.* are in the *Carteſian* Syſtem but the Effects of Compreſſion every Way. Wherefore theſe Parts will be divided at the leaſt Shock, in the ſame manner as if Quickſilver be thrown againſt a Wall, it is inſtantly ſeen to be divided into a Million of Parts, to be reflected on every Side, and be again divided as ſoon as it falls on the Floor. I know my Compariſon is not exact, but the Advantage is on my Side; becauſe Quickſilver is not without Viſcoſity, or a certain Tenacity between its Parts; whether it proceeds from Attraction, which is my Opinion, or that it be the Effect of the Preſſure of the ambient Fluid. Therefore the *Carteſian* Matter will have more Facility to divide than Quickſilver, and will not be ſuſceptible of any regular Motion; which alone demonſtrates, that the mechanical Generation of the *Vortex* is impoſſible.

There is however this Difference between the *Vortex* imagined by *Deſcartes*, compoſed of hard Globules; and that of the infinitely ſoft Matter of Father *Malebranche*, whoſe Syſtem is revived by his Diſciple Monſieur *de Molieres*; that if the *Carteſians* ad-

admitted Gravity as a Principle; beſides that it would give the true Cauſe of Hardneſs, its Combination with the ſtrait or projectile Motion would produce a Motion in a Curve; as Sir *Iſaac Newton* has demonſtrated. But until they will return to this Idea of primitive Gravity, and further while they will make uſe of no other Matter than one infinitely ſoft, and really unintelligible, it will not be poſſible to conceive a ſingle *Vortex* formed; far from having this infinite Number, which, by-the-bye, ought to be diſſipated as Waves raiſed in the Water, upon account of their perfect Homogeneity.

The famous *Carteſians*, always refuſing to allow this primitive Gravity, and at the ſame time plainly ſeeing, that this firſt Manner of forming the *Vortex* was impoſſible, have had recourſe, in order to its Formation, to the Motion of Rotation of a ſolid Sphere at the Centre of a ſmall Particle of Matter at Reſt, *&c.* and they have pretended, that this Sphere in its Circulation ought to carry along with it the circumambient Matter.

But this Notion is certainly as unſuſtainable as the Firſt. For,

1*ſt*, They muſt explain to us the mechanical Formation of this Sphere; they muſt account for its Solidity: But all this manifeſtly ſuppoſes the *Vortex* already formed; all this ſuppoſes a Preſſure equal on every Side, uniform and concentric.

2*dly*, This Sphere would never imprint an equal Velocity on all the Points of the concave Surface which touches and incloſes it, ſeeing itſelf has not an equal Velocity in every Point of its laſt Surface; and therefore the *Vortex* would not have as much

Force to defend itſelf towards the Poles, as towards the Equator; as we ſhall ſhew hereafter.

3*dly*, This Sphere, in ſtriking againſt the ambient Matter, would but divide it *ad infinitum*; becauſe it is infinitely ſoft, and that its Parts have no Adherence with each other.

4*thly*, It is not ſufficient, that a Sphere turns round its Centre, to draw into its Circulation the ambient Matter: It is moreover requiſite, that to preſs on this Matter in a Direction from the Centre to the Circumference, (which a ſolid Globe either cannot do, or can hardly be conceived poſſible for it to do) and further ſtill, it is neceſſary there ſhould be Unevenneſſes on this Sphere, and on the concave Surface of the ambient Matter; becauſe otherwiſe, though the Sphere ſhould preſs this Surface by its centrifugal Force, it would only raiſe it up, or tend to raiſe it, and it would ſlide along the Surface without dragging it away with it: On which Head there is this Particularity to be remarked, that, for the uniform Circulation and Conſervation of the *Vortex*, and ſtill more for the preſerving of *Kepler*'s Laws, the Spheres and Surfaces muſt be ſtrictly Mathematical, as we ſhall ſoon ſee; and for its Formation they muſt be rough, and full of Unevenneſſes: But what can be more whimſical? And further, though theſe Surfaces were full of Prickles, yet could not the *Vortex* be formed in the Hypotheſis of Father *Malebranche*'s ſoft Matter; becauſe the Parts which would form theſe Eminences and Unevenneſſes on the concave Surface of the Matter ſurrounding the Sphere, not being connected with the other Parts of the ſame Matter, would be carried off without Difficulty by the Rotation of the Sphere; and the reſt of the Matter would remain at Reſt. And thoſe who would pretend, that theſe Unevenneſſes, theſe Parts which form the Hillocks we are ſpeaking of, could not, in conſequence of GOD's Decree, looſe themſelves from the other Parts of the Matter, would evidently abandon Mechaniſm, without reaping any Advantage: Becauſe, ſuppoſing it true, that by this Means the ambient Matter would be compelled to circulate, yet could it not form a fluid *Vortex*, wherein *Kepler*'s Laws could be obſerved; becauſe both the Sphere and theſe Surfaces being by theſe Unevenneſſes wedged into each other by ſolid hard and inflexible Parts, they would neceſſarily move all of a Piece, as the Parts of a Sphere do.

5thly, By means of this Sphere one could have but a great *Vortex* formed; and not that infinite Multitude of ſmall *Vortices*, with which the great ones are at this Day ſuppoſed to be filled, and in the Centre of all, or moſt Part of which, People will not allow that there are hard Globules, and ſo of the reſt: For I am perſuaded, that the Reader, by a little Meditation on this Subject, will find almoſt as many Reaſons againſt this Syſtem, as there are ſmall *Vortices* ſuppoſed to exiſt.

It may be objected, that we do not pretend to form a *Vortex*: We ſuppoſe that GOD formed it in the Beginning, and in Conſequence hereof we account for its Properties and Conſervation.

But, beſides that the Impoſſibility of the mechanical Generation of a *Vortex* is a ſtrong Prejudice againſt its Conſervation; I pretend, in the Principles of our Adverſaries, GOD could not form a ſingle *Vortex*.

I deſire Attention may be given, that a circular Motion is a redoubled and forced Motion; and not, as Mr. *Perault* thought, a natural Motion. Now the rectilinear Motion cannot be redoubled thus, as againſt its Nature, in order to become circular, but upon a Suppoſition that it meets in the ambient Matter invincible Obſtacles to its Direction; or that by a primitive Law it is carried towards a Centre by a Motion of Gravitation, at the ſame time that it receives a Motion in a ſtrait Line. Therefore, ſince on one hand this univerſal and primordial Gravity is obſtinately rejected; and on the other, as it is ſolidly proved above, that the ambient Matter is no Obſtacle to the rectilinear Motion; it remains certain, that the Formation of the *Vortex* is impoſſible. *Q. E. D.*

SECOND *and* THIRD PART.

The Vortex, *though once formed, cannot laſt, and it is not ſufficient for explaining the celeſtial* Phænomena.

POSTULATUM.

The cylindric *Vortex* cannot long ſubſiſt, and is not ſufficient for explaining the celeſtial *Phænomena*: This Principle is allowed by all *Carteſians* in both its Parts. It cannot ſubſiſt; becauſe not having Force to defend itſelf towards the Poles, if it happened to hit on that Side againſt another cylindric *Vortex*, that preſented its Equator, it would ſoon be broke into, and burſt to its very Centre. If, on the contrary, its ſame Side touched another cylindric *Vortex* by the Poles, they would both mix together, and would compoſe but one *Vortex*.

It is not ſufficient for explaining the celeſtial *Phænomena*; becauſe it is allowed, that the tranſlative Velocities of its Points cannot be in an inverted *Ratio* to the Roots of the Diſtances, and that its centrifugal Force does not diminiſh in the inverted *Ratio* of the Squares of theſe Diſtances, *&c.*

COROLLARY.

Therefore the ſpherical *Vortex*, in order to be of Uſe, muſt have other Properties than the cylindric: That is to ſay, it muſt have a relative Force to one and the ſame Centre; for it is by this Force alone that it can be different from the cylindric *Vortex*.

This Force, moreover, muſt be equal in all the Points of the ſame ſpherical Superficies; becauſe otherwiſe it might be burſt and broke into in its weak Parts, as well as the cylindric, *&c.*

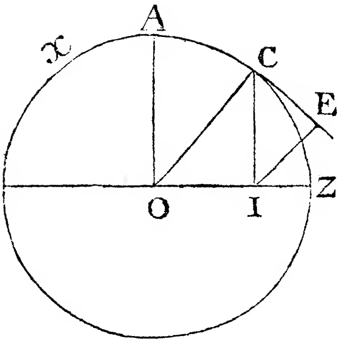
THEOREM I.

Even in the ſpherical *Vortex* there is no relative Force to one and the ſame Centre: That is to ſay, that it has properly but an axifugal Force.

DEMONSTRATION.

The ſpherical *Vortex* is compoſed, as well as the cylindrical, of ſeveral parallel Circles, but with this Difference, that in the ſpherical *Vortex* the *Radii* of the parallel Circles are not all equal, but on the contrary diminiſh according as they recede from the Equator, and approach the Poles. Now it is manifeſt, that all the parallel Circles circulating round different Points of the Axis in the ſpherical *Vortex*, as well as in the cylindrical, tend to recede only from theſe different Points of the Axis, round which they circulate; becauſe a Body cannot tend to recede from any Centre but that of its Circulation. In a Word, in order to make a *Vortex* ſpherical, which was cylindrical, they have but proportionally ſhortened the parallel Circles. But let the *Radius* of a Circle be ever ſo much ſhortened or lengthened, that will not change the Direction of its dilatative Effort. I am miſtaken! an imaginary Line is going to change the Direction of the axifugal Force. This Force, as all agree, has for its Direction the *Radius* I C, in the Circumference whereof it is the *Radius*; but the Direction I C is oblique to C E the Tangent to the Sphere; therefore it changes, according to the general Law of an oblique Shock, into the Determination I E or O C relative to the Centre O.

But if Lines may be imagined, and that nothing more is requiſite to realize them, than Points that correſpond to them; we ſhall have ſome of all ſorts in the *Vortex:* We ſhall have oblique Lines on the *Radius* O A, a perpendicular one, and ſome more or leſs oblique, on the *Radius* I C, and by that means we ſhall be able to determine nothing. Let us grant however, that there is a Tangent to the Sphere C E, at the Point C, and let us ſee if it will be a ſufficient Reaſon for decompoſing the centrifugal Force I C into a central Force I E or O C. For that Purpoſe I ask, What are the Points that compoſe this Tangent?



gent? It is evident, that it can only be the Globules of the upper *Stratum* that anſwer thereto. The Line CE is therefore compoſed only of a certain Number of Points ſeparate one from the other, and which conſequently can move one without the other. Therefore if the Line IC is perpendicular to the Globule that occupies the Point C, and that it paſſes through its Centre; there will be no Decompoſure, and the Force IC will not change into a Force that has the *Radius* OC for its Direction.

Now it is infinitely probable, that the *Radius* IC paſſes through the Centre of the Globule C; and it is eaſy to demonſtrate, that it is actually ſo even in the Principles of Monſieur *Saurin*, who firſt invented this central Decompoſition. For what has been the Cauſe of the Decompoſition of the circular Velocity into the centrifugal Force IC? It ſeems plain to me, that no other Cauſe can be aſſigned than the Point or Globule C; ſeeing there is but that one at the Point where it happened. The Line IC paſſes then through the Centre of the Globule C; ſince the De-

Decompoſition is always made in a perpendicular Line to the Point that cauſed it.

And indeed, either the *Radius* IC paſſes through the Centre of the Globule C, or the Centre of this Globule is on one Side or the other of this *Radius*, but ſo as that this *Radius* cuts the Point C; or elſe, it is a Space intercepted between Two Globules, which directly anſwers to the Point C. In the Firſt Caſe, there is no Decompoſition: In the Second, and in the Hypotheſis, that the Centre of the Glòbule C happens to be between the *Radius* IC and the Equator, there will be a Decompoſition; but it is manifeſt, that it will not be a central one: It will, on the contrary, be relative either to the very Pole, or to one of the polar Circles. In the Third Caſe, wherein it is ſuppoſed, that it is a Space intercepted between Two Globules, which anſwers to the Point C; there may be a Decompoſition, but it will be double, the one relative to the Centre O, and the other relative to the Pole Z.

Now the *Carteſians* can never draw from this Decompoſition the Advantage they propoſe; becauſe there will not be more Reaſon for heavy Bodies precipitating to the Centre of the Sphere by means of the central Force, than to the very Pole by the Aſſiſtance of the polifugal Force; or rather, the Complication of theſe Two Forces will compel the *Mobile* to precipitate to the Centre I of the Parallel it happens to be in.

Wherefore, in order to defend the ſpherical *Vortex*, they muſt ſay, that the Centre of the Globule C is comprehended between the Poles and the *Radius* IC. But on what Foundation will they aſſure it?

What are the Proofs they will give for it? One muſt certainly be a very bold Gameſter, to hazard this Point; becauſe beſides the Appearance of Truth, the Adverſaries of *Vortexes* may wager Three to One, that it does not ſo happen. But in caſe it be allowed, will they ever find in the ſoft Matter of Father *Malebranche* and Monſieur *de Moliere*, a ſufficient Cauſe of the Decompoſition? There muſt be a Reſiſtance to produce a Decompoſition, and an infinitely ſoft Matter does not reſiſt. And further, in the Hypotheſis of the Decompoſition of I C or O C, the *Vortex* would not be in Safety; becauſe there would be a Remainder of the centrifugal Force IC, that would be parallel to the Tangent C E, and would evidently ſpread Confuſion in the *Vortex*, by driving all the parallel Circles towards the Equator.

This ſeems to me ſufficient to diſcredit, in the Minds of rational People free from Prejudice, this central Force, which is attempted by all means to be introduced. But let us not be tired of examining this Point thoroughly: It is of Conſequence, and the *Carteſians* well deſerve the Trouble of an abundant Refutation. Wherefore let us ſuppoſe, that GOD forms a *Vortex* cylindrical and fluid; it is a received and evident Principle, that its Points will have but an axifugal Force. And if a Sphere be conceived to be inſcribed in this Cylinder, the Points that compoſe it, will not in like manner have any central centrifugal Force, according to the Axiom: *Noſtrum intelligere nihil ponit in re.* Now let us realize this ſpherical *Vortex*, which before we had but conceived; that is, let us ſuppoſe, that GOD has deſtroyed the tranſlative Velocity of the Points that form the

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angular Spaces intercepted between the laſt Surface of the inſcribed Sphere and that of the Cylinder; it is manifeſt, that no Change will happen in the Velocity and axifugal Force of the reſt of the Points, which are not included in theſe; for this Reaſon, that the Points which fill the Two kinds of Baſons that mark the Exceſs of the Cylinder above the inſcribed Sphere, remain in the ſame Order, Diſpoſition, and Direction, with regard to the inferior Points, which they were in at the time of their Motion. And there is no other Difference to be perceived herein, except that at preſent it is the ſame Point that conſtantly correſponds to the ſame Place; and that before this Place was ſucceſſively occupied by Points intirely reſembling each other, and that which remains or is ſuppoſed conſtantly to remain therein.

Now whether this Place be conſtantly occupied by one and the ſame Point, or ſucceſſively by Points intirely alike and in the ſame Order, is what ought not to produce any Variation in the Effect which we are examining: And this appears to me at leaſt as clear as Noon-day.

Wherefore, ſince theſe inferior Points had then but an axifugal Force, it follows that even now they have no other Tendency than to recede from the Centres of their Circulations, without having any Force relative to the Centre of the *Vortex*.

This is all that pure Reaſon dictates to me on this Point of the Nature of the *Vortex*, whether ſpherical or cylindrical: And I dare flatter myſelf, that whoſoever will attentively examine my Reaſonings, will find them as demonſtrative as can be deſired in Natural Philoſophy.

In Effect, Experience agrees here with Reaſon. If a glaſs Globe filled with Water be rapidly turned on its Axis, one ſees little Foulneſſes; the ſmall Atoms which it never fails to contain, gather together along the Axis, and form a little Cylinder round it.—— Which very plainly ſhews, that in this ſpherical *Vortex* of Water there is but an axifugal Force. *Q. E. D.*

COROLLARY.

Therefore Gravity is inexplicable in the *Vortex*, and it has not Strength to defend itſelf towards the Poles.

THEOREM II.

Suppoſing there was in the ſpherical *Vortex* a central Force according to the *Radius* O C, it could not by Reaction be changed into a centripetal Force according to the *Radius* C O.

This Propoſition is well known to all who are ſomewhat converſant in Mechanics.

It is therein demonſtrated, that if the *Radius* I C, for Example, forms with the Tangent C E an Angle of 45 Degrees, the Line of Reflexion will be parallel to the Axis; and that from the Point C to the Pole Z, the Lines of Reflexion will be divergent to the Axis; and, in fine, that from the Point C to the Equator, theſe ſame Lines of Reflexion will be indeed convergent to the Axis, but will never terminate at the Centre O: In a word, that becauſe the Angle of Reflexion is always equal to the Angle of Incidence, it is only at the Equator that the centrifugal Force can be changed into a centripetal Force. *Q. E. D.*

COROLLARY I.

Therefore the modern *Carteſians* are ſtrangely miſtaken, when they pretend to account for Gravity by the Reverſe of the central centrifugal Force.

COROLLARY II.

And they can never, *à fortiori*, in their Principles, explain the Figure of the Earth and of *Jupiter*, which are flatted Spheroids made by the Converſion of an Ellipſis upon its ſmall Axis.

LEMMA I.

If the centrifugal Force repreſented by I C (ſee *p.* 420.) be decompoſed on the ſpherical Tangent into a Force, that for its Direction has the Centre of the Sphere; the central Force, which reſults from this Decompoſition, will be to the centrifugal Force, as the *Radius* I C to the *Radius* O C.

For the centrifugal Force I C, being decompoſed into C on the Tangent of the Sphere, will ſtrike this Tangent with a Force that will be repreſented by I E. But on account of the ſimular Triangles I E C, I O C; IE. I C : : I C. O C.

LEMMA II.

A Body which deſcribes a Curve, ſtrikes this Curve every time it paſſes from one Side to the other, with an infinitely ſmall Force of the firſt kind with regard to its Velocity.

To the beſt of my Remembrance, this Propoſition is demonſtrated in Dr. *Clarke's* Notes on *Rohault's Phyſica*, and in Monſieur *de Moliere's* Lectures: And it is evident from this alone, that it can only be by a Force repreſented by the Sine of the Angle of Contact that this moveable Body ſtrikes the Tangent of its Curve.

THEOREM III.

Let us put Complaiſance on the Stretch, and grant that *Vortexes* have a central and centripetal Force relative to one Centre O: I ſay, that the ſpherical *Vortex* will not have as much of this central Force, to defend itſelf towards the Poles, as towards the Equator.

CONSTRUCTION.

Let us take, in the ſame Superficies X (ſee the Fig. *p*. 420.) Two Points at Pleaſure, the Point A in the Circumference of the Equator, and the Point C in the Circumference of a ſubduple parallel Circle; we will give in the Demonſtration an equal Velocity to the Globules which circulate in theſe Two Circumferences; which is the moſt favourable Conceſſion imaginable for the Patrons of *Vortexes*.

DEMONSTRATION.

It is manifeſt, that if the Point A is in an equal Space of Time ſtruck an equal Number of Times as the Point C, and that each Stroke againſt the Point A be double each Stroke againſt the Point C; it is manifeſt, I ſay, that there is more Force at the

Equator than at the parallel Circle. Now the Suppoſition is very certain in both its Parts: For,

1. Since the Circumference of the Equator is the double of that of the parallel Circle, and that being at an equal Diſtance from the Centre O, (ſee Fig. p. 420.) the Globules they contain are equal to each other; if there be a Thouſand Globules in the Circumference of the Parallel, there will be Two thouſand in the Circumference of the Equator. And as theſe Globules are ſuppoſed to have in both an equal Velocity, they will make (but) One Revolution in the Equator, while thoſe of the ſubduple Circumference will make Two. Therefore, in both, there will be Two thouſand Strokes employed in the ſame Space of Time, againſt the Points A and C.

2. Each central Stroke is double at the Equator: Becauſe, as there is in both an equal Velocity, and that (Lem. II.) each centrifugal Stroke in every Circumference is a Fluxion of the firſt kind, with regard to the Velocity of the Globule which is in Motion; it follows that the centrifugal Strokes both in the Equator, and in the parallel Circle, are equal to each other. But the central Effort (which is the only one by which a *Vortex* can defend itſelf towards the Poles) is at the Point C (Lem. I.) but half the centrifugal Effort, ſince it is repreſented by IE ſubduple of IC; whereas at the Equator the central Effort is the ſame with the centrifugal Effort, becauſe the *Radius* OA is perpendicular on the ſpirical Tangent, which correſponds to it. Therefore, *&c.* *Q.E.D.*

COROLLARY I.

Therefore if a *Vortex* be in *Æquilibrium* with another *Vortex*, and that the Equator of one happens to anſwer to the Poles or Tropics of the other; the latter will be burſt and penetrated to the Centre: And I do not think, that the *Carteſians* can find their Account in this Conſequence.

COROLLARY II.

Therefore if the *Vortex* was the mechanical Cauſe of Gravity, Gravity ought to be greater at the Equator than at the Poles; and the Earth would be an oblong Spheroid; which is contrary to Obſervations.

REMARK.

I have ſaid, that it was making a large Conceſſion to the *Carteſians*, to ſuppoſe that the Globules of both the Circumferences have an equal Velocity. For if a Sphere full of Water be made to turn on its Centre, Experience teaches, that the Velocity is greater at the Equator than in the parallel Circles; ſince it is obſerved, that the Times of their periodical Revolutions are equal. Whence it follows, that I have, in my Demonſtration, made the moſt favourable Suppoſition for the *Carteſians* that was poſſible.

THEOREM IV.

In order to determine the Tendency of a Layer towards the Upper Part of the *Vortex*, regard muſt be had not only to that which reſults from its own Circulation, but alſo to that which it receives from the other lower Layers, unleſs it be the Layer next the Centre.

DEMONSTRATION.

While a Layer is in Circulation, it viſibly makes a continual Effort towards dilating itſelf, by reaſon of the centrifugal Force, with which all its Parts endeavour to recede from the Centre of Circulation: But its actual Dilatation being impeded by the Layer next above it, this laſt will be naturally preſſed by it. And thus it is that the firſt or loweſt Layer, being put into Circulation, preſſes the Second; and the Second, aſſiſted by the Firſt, preſſes the Third; this, aſſiſted by the Two preceding, preſſes the Fourth; and ſo on from Layer to Layer, through the whole Extent of the *Vortex*. Whence it follows, that in order to eſtimate the Quantity of Force with which a Layer tends towards the Surface of the *Vortex*, one muſt take the centrifugal Force proper to this Layer and that, which all the Matter of the Fluid contained under it acquires by Circulation. *Q. E. D.*

COROLLARY I.

Therefore the dilatative Effort of the Layers increaſes with the Layers in a greater Proportion than theſe Layers.

COROLLARY II.

Therefore it is impoſſible to explain in the *Vortex*, how Gravity decreaſes in an inverted *Ratio* of the Squares of the Diſtances; and conſequently there will be nothing found in the *Vortex* to anſwer to

Attraction, whoſe Exiſtence Sir *Iſaac Newton* has ſo demonſtratively eſtabliſhed.

COROLLARY III.

Thus we have re-eſtabliſhed in its full Light the Difficulty, which Monſieur *de Fontenelle* propoſed to Monſieur *Villemot* in the Memoirs of the Academy for the Year 1705 *. This learned Academician pretends, that as in the *Vortex* the lower Points ought to move faſter than the upper, in order to preſerve *Kepler's* Aſtronomical Law; they ought alſo to have a greater centrifugal Force, and conſequently compel them to deſcend, particularly in proportion to their Fluidity. The Objection made a great Noiſe, and the only Method found of getting rid of it, was by ſaying, that although each lower Point had more centrifugal Force than each upper; yet as the *Vortex* was in *Æquilibrium*, and the Sums of the Force of each of the Two Layers were equal, there was no Reaſon why the lower *Stratum* ſhould get the better of the upper; becauſe this was as prevalent by the Number of its Points, as that was by the Force of each of its own.

But it is manifeſt, after what has been demonſtrated above, that the ſecond Layer, being aſſiſted by the firſt, muſt have a greater Force than the third, and conſequently compel it to deſcend, purſuant to the Principle then granted to Monſieur *Fontenelle*.

But if it be asked, How could the upper Layer deſcend, ſeeing Matter is impenetrable?

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* He afterwards publiſhed a Book, intituled, *Nouveau Syſteme, ou nouvelle Explication du Mouvement des* Planetes, *par M.* Philippe Villemot, *Pretre, Docteur en Theologie*, &c. Lyon, 1707. in 12°.

I ſhall ask in my turn, How, in an intire *Plenum*, do heavy Bodies fall to the Centre? And I reaſon on the Principle granted to Monſieur *Fontenelle*.

But yet, becauſe what is allowed by one *Carteſian* is not always allowed by all; let us ſuppoſe, that the upper Layer cannot deſcend; this, at leaſt, will follow from my Demonſtration, that, according to the Principles of all theſe Gentlemen, an upper Layer being preſſed by all the under ones, it muſt haſten its Circulation, as long as it is ſlower than that of theſe under Layers; by reaſon that the Exceſs of their Velocities will act upon it, as if it had been at Reſt.

COROLLARY IV.

Therefore the Layers of a *Vortex* will move all of a Piece, as do thoſe of a ſolid Sphere; and *Kepler's* Law cannot poſſibly be preſerved. We ſhall now give other Proofs upon other Principles.

THEOREM V.

The Motion of the Points of the Equator is abſolutely independent of the Motion of the parallel Circles; and conſequently, in order to determine the *Æquilibrium* of the Points of the Equator, we muſt attend to nothing but its Motion.

DEMONSTRATION.

The Plane of the Equator is parallel to the Planes of the other parallel Circles, that turn round the ſame Axe with it: Its centrifugal Force is perpendicular to the Tangent to the Sphere, which anſwers to it: It has not then any lateral Tendency towards theſe parallel Circles, and by a neceſſary Conſequence its Motion is abſolutely independent of theirs.

And indeed, if it be ſuppoſed, that the Motion of the other parallel Circles ſtops, there is ſtill ſome Motion conceived in the Equator, juſt as in the Caſe of the cylindrical *Vortex:* It is likewiſe conceivable, that the Velocity may be greater at the Equator than in the parallel Circles, as the Experiment already cited ſhews us: And if no Regard be had to the lateral Frictions, as the *Carteſians* would have it, who ſuppoſe them none or inſenſible, and as indeed they are obliged to ſay, that the *Vortex*, by the lateral Friction of the Equator, may not become cylindrical; this Equator will always continue to circulate uniformly, without communicating any of its Velocity to the Points that laterally ſurround it. Therefore, *&c.* *Q. E. D.*

COROLLARY I.

Therefore for the *Æquilibrium* of the Points of the Equator, it is neceſſary, at leaſt, that an upper Circumference ſhould have as much Tendency towards the Superficies of the *Vortex*, as another under concentric Circumference; becauſe, if it had leſs, there would be no *Æquilibrium*, even in the Principles of the *Carteſians*; and the under Circumference, preſſing the upper, would either make it deſcend, or communicate to it a Force equal to its own. Wherefore, calling F the proper centrifugal Force of a Point of the upper Circumference, and f that of a Point of the under one; if S, s mark the different Sums of the Points contained in theſe Two Circumferences, we ſhall have $FS = fs$.

COROLLARY II.

Therefore the centrifugal Force does not diminiſh in the Plane of the Equator in the inverted *Ratio* of the Squares of the Diſtances from the Centre; for ſince $FS=fs$; $F.f :: s.S$. But the Points being ſuppoſed equal on both Sides, their Sums are as the Circumferences, and one has $s.S. :: d.\mathcal{D}$, which gives $F.f :: d.\mathcal{D}$. inſtead of $:: dd.\mathcal{D}\mathcal{D}$.

COROLLARY III.

Therefore *Kepler*'s Rules cannot be obſerved in the *Vortex*, or at leaſt in the Plane of its Equator; for ſince $F.f :: d.\mathcal{D}$; by putting in the Place of F,f, their Values, we ſhall have $\frac{VV}{D}.\frac{uu}{d} :: d.\mathcal{D}$, and therefore $V=u$ and $\mathcal{D}^2,d^2 :: TT.tt$. whereas we ought to have $V.u :: \sqrt[2]{d}.\sqrt[2]{D}$ and $\mathcal{D}^3.d^3 :: TT.tt$.

REMARK.

There is here a *Fineſſe* of the *Carteſians* to be obſerved. Theſe Gentlemen conſider only the *Æquilibrium* of the ſpherical Layers of the *Vortex*, and from the Equality of their central Forces they deduce *Kepler*'s Laws, as well as they can.

But it is manifeſt, that whatever becomes of the Equality of Force in different ſpherical Superficies of the *Vortex*, there muſt be an *Æquilibrium* in the Plane of the Equator; becauſe it is in this Plane that the Planets move; and if there had not actually been an *Æquilibrium* between its Points, they would ſoon place themſelves there, by reaſon that Fluids always tend to the Side where they are leſs preſſed; and it is by an actual *Æquilibrium* alone that they are kept in their Places; which intirely overturns the Theory of theſe Gentlemen.

Let us however grant to the *Carteſians*, that the Sums of the Forces of the Two ſpherical Surfaces are equal; I cannot ſee, that they can thence infer, as they do, that the central Force diminiſhes in a reciprocal *Ratio* of the Square of the Diſtance from the Centre. Let us examine their Argument:

$FS = fs$, ſay they; therefore $F.f :: s.S$; but s, S mark the Sums of the Points contained in the Two Surfaces; therefore they are as theſe Surfaces, which, being as the Squares of their Diſtances, give, $F.f :: dd.DD$.

But it muſt be remarked, that the Surfaces of the *Vortex* are not Mathematical, they are Surfaces which have ſome Thickneſs: They cannot then be proportional to the Squares of their Diſtances from the Centre, except in the Caſe when their Thickneſs is equal. Now, according to the *Carteſians*, the Points or Globules, which compoſe the *Vortex*, increaſe in Bulk according as they recede from the Centre; and, beſides, they are homogeneous, or of an equal ſpecific Denſity, at leaſt in their common Syſtem. And conſequently it is certain, that the different natural or real *Strata* of the *Vortex* are not of an equal Thickneſs, and that the Matter contained therein is not proportionate to the Squares of the *Radii* of theſe Surfaces, but only to the Squares of theſe *Radii* multiplied by the Thickneſs of the *Strata*. Therefore, *&c.* *Q. E. D.*

COROLLARY IV.

Therefore, even allowing the *Carteſians*, what one has a Right (COROL. I. THEOR. IV.) to refuſe them, they will never be able to explain *Kepler's* Rules in the *Vortex*; for it is only by the Proportion, which I have juſt now annulled, that they pretend to do it. See M. *de Molieres's Leçons de Phyſique*.

And if it be objected, that I have not, in the preceding Corollaries, had any regard to the Thickneſs of the Circumferences; I anſwer, that it was by way of pure Conceſſion that I have not done it; and if any Perſon will be at the Pains of doing it, he will eaſily find, that *Kepler's* Rules will only be the more diſturbed thereby.

CONCLUSION.

Therefore the *Vortex* is every way impoſſible, and inſufficient in Natural Philoſophy. Its mechanical Generation is impoſſible (PART I.); it has only an axifugal Force, and not a centrifugal and centripetal Force, as it ſhould have (THEOR. I. and II.); and even if it had, it cannot (THEOR. III.) defend itſelf equally on all Sides. It is not ſufficient for explaining Gravity, and its Properties; it deſtroys *Kepler's* Aſtronomical Laws (COROL. III. THEOR. IV. and V.). What more can be deſired, in order to conclude with Sir *Iſaac Newton?* "Itaque hypotheſis Vorticum "(eſt impoſſibile &) cum phænomenis aſtronomicis "omnino pugnat, & non tam ad explicandos quam "ad perturbandos motus cœleſtes conducit." *Q.E.D.*